A Formal Description for Hierarchical Coalitions

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The definition of hierarchies as organisational structures for coalitions is the first step toward the definition of a joint human-agent planning framework. However, a more formal description of such structures is important to be used as a basis for future discussions, so that ideas can be introduced on a same perspective. Figure 2.4 illustrates the idea of a general hierarchy.

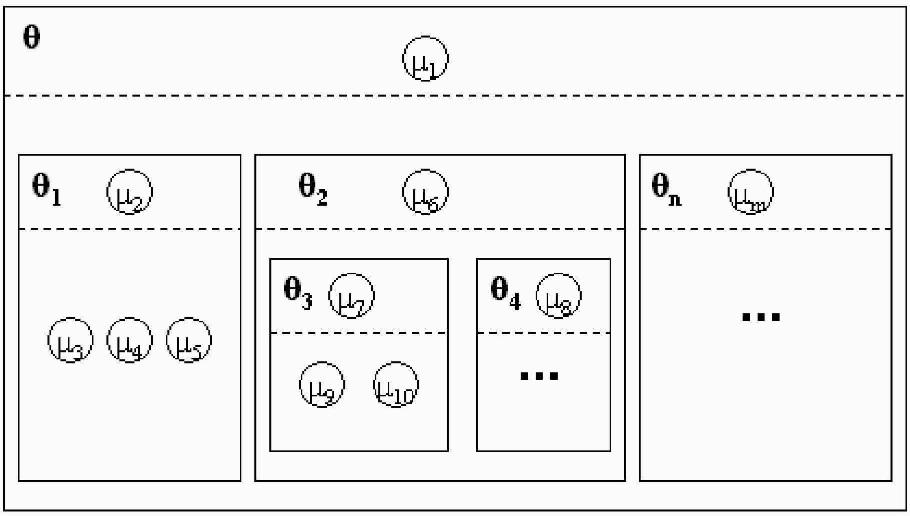

Figure 1: Example of a hierarchical coalition description

Components (agents) that form a hierarchy are represented by μ_i, where i is an integer value from 1 to n (total number of components). We can define the following functions on hierarchical components:

- LEVEL(μ_i), returns the level of μ_i. The notion of levels is introduced through the idea that components at the same depth belong to the same hierarchical level;
- RELATION(μ_i,μ_j), returns the relation of μ_i regarding μ_j. If such components do not have a relationship, the function returns null.

Using such functions we can deduce some initial properties. First, considering two different components μ_i and μ_j, if μ_i is peer of μ_j, then they are at the same level. However the return is not true because components in the same level can have a null relationship. Such a property can be expressed as:

$$\forall\ \mu_i, \mu j\ (i \neq j)\ \wedge\ RELATION(\mu_i, \mu_j) = \text{ peer}$$
$$\Rightarrow\ LEVEL(\mu_i) = LEVEL(\mu_j)$$

In the same way we can deduce properties for the cases where μ_i has a superior or a subordinate relation regarding μ_j. Note that in such cases μ_i and μ_j have to be in adjacent levels (we assume the highest level as level 1).

$$\forall\ \mu_i, \mu j\ (i \neq j)\ \wedge\ RELATION(\mu_i, \mu_j) = \text{ superior}$$
$$\Rightarrow\ LEVEL(\mu_j) - LEVEL(\mu_i) = 1$$

$$\forall\ \mu_i, \mu j\ (i \neq j)\ \wedge\ RELATION(\mu_i, \mu_j) = \text{ subordinate}$$
$$\Rightarrow\ LEVEL(\mu_i) - LEVEL(\mu_j) = 1$$

We are assuming that components can only set relations with components of their level or adjacent levels. Thus, the difference between their levels is 0 or 1:

$$\forall\ \mu_i, \mu j\ (i \neq j)\ \wedge\ RELATION(\mu_i, \mu_j)\ \neq \text{Null}$$
$$\Rightarrow\ |LEVEL(\mu_i) - LEVEL(\mu_j)|\ \leq\ 1$$

Relationships inside a coalition are always between two components. Each relationship also defines a communication channel between the components so that they can exchange useful messages for the performance of their plans. Messages can be represented by the tuple $< \mu_i, \mu_j, content >$, where μ_i and μ_j are the message sender and receiver respectively, and *content* could be instances of commands, goals, activities, feedback, facts and so on. The kind of relationship between μ_1 and μ_2 has influence on this communication, enabling or avoiding the sending of some types of message. For example, components that have a peer-peer relationship may not be able to exchange commands between them.

An option to describe a hierarchical coalition Θ is to consider Θ a composition of sub-coalitions. To this end, we can use the tuple $< \mu_i, S_{[1..m]} >$, where μ_i is a superior agent and $S_{[1..m]}$ is a set of subordinates that can be formed by components ($\mu_{[1..m]}$) or sub-coalitions ($\Theta_{[1..m]}$). In this last case, each Θ_i can recursively be decomposed in their components or sub-coalitions. For example, to represent the hierarchy of Figure 2.4 we have:

$$\begin{aligned} \Theta &= < \mu_1, [\Theta_1, \Theta_2, \Theta_n] > \\ &= < \mu_1, [< \mu_2, [\mu_3, \mu_4, \mu_5] >, < \mu_6, [\Theta_3, \Theta_4] >, < \mu_m, [...] >] > \\ &= < \mu_1, [< \mu_2, [\mu_3, \mu_4, \mu_5] >, < \mu_6, [< \mu_7, [\mu_9, \mu_{10}] >, < \mu_8, [...] >] >, < \mu_m, [...] >] > \end{aligned}$$

Another practical way to represent sub-coalitions is to use the concept of *interaction zones*. Each interaction zone Φ_i defines a group of agents that present a direct communication between them. For example, in Figure 2.4 we could define six interaction zones with their respective agents: $\Phi_1 = \{\mu_1, \mu_2, \mu_6, \mu_m, \}$, $\Phi_2 = \{\mu_2, \mu_3, \mu_4, \mu_5\}$, $\Phi_3 = \{\mu_6, \mu_7, \mu_8\}$, $\Phi_4 = \{\mu_7, \mu_9, \mu_{10}\}$, $\Phi_5 = \{\mu_8, ...\}$ and $\Phi_6 = \{\mu_m, ...\}$. Note that the sets of agents in each zone Φ_i are always represented by one superior and one or more subordinates. In this way, the tuple $< \mu_i, S_{[1..m]} >$ can be applied to represent such sets as sub-coalitions. Considering this idea, we have the following sub-coalitions for each interaction zone: $\Theta_{\Phi_1} =< \mu_1, [\mu_2, \mu_6, \mu_m] >$, $\Theta_{\Phi_2} =< \mu_2, [\mu_3, \mu_4, \mu_5] >$, $\Theta_{\Phi_3} =< \mu_6, [\mu_7, \mu_8] >$, $\Theta_{\Phi_4} =< \mu_7, [\mu_9, \mu_{10}] >$, $\Theta_{\Phi_5} =< \mu_8, [...] >$ and $\Theta_{\Phi_6} =< \mu_m, [...] >$. In brief, a general rule for a coalition $\Theta =< \mu_i, S_{[1..m]} >$ is:

IF $S_{[1..m]} = \mu_{[1..m]} \;\Rightarrow\; \Theta_\Phi =< \mu_i, [\mu_1, ..., \mu_m] >$
IF $S_{[1..m]} = \Theta_{[1..m]} \;\Rightarrow\; \Theta_\Phi =< \mu_i, [Superior(\Theta_1), ..., Superior(\Theta_m)] >$

Using such a definition we can consider that a coalition has a number of interrelated sub-coalitions that are themselves hierarchically structured. Each sub-coalition is a stable intermediate form and can most of the time act without help from the complex structure. At this point we can apply the following function to return plans from a (sub)coalition:

- PLAN(Θ_i,p), returns the (sub)plan of a (sub)coalition Θ_i to a proposition p. The same function can be applied to return the plan of a component μ_i.

Plans are intricately linked to the idea of levels so that components on the same level share a common degree of plan abstraction. The following property can be defined to relate plans of an upper level component with the plans of their subordinates:

$$\forall < \mu, S_{[1..m]} > \;\; PLAN(\mu, p) \;=\; \bigcup_{i=1}^{m} PLAN(S_i, p_i)$$

This property is important to corroborate, for example, the idea of enclosing planning problems inside the sub-coalition where they were generated. In this way, if PLAN($< \mu, s_{[1..m]} >$) has a problem, μ must deal with such a problem together with its subordinates $S_{[1..m]}$. Only if this is not possible, μ will report the problem to its superior.